Tutorial-1

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* Trace the curve

① $xy^2 = a^2(a-x)$

⇒ ① Symmetric :-
The curve is symmetric about x-axis.

② origin :- Put $x = y = 0$,
So we get $0 = a^3$

$0 \neq a^3$ so, The curve does not passes through the origin

③ Intercept :-
Put ⇒ $y = 0$
$\therefore 0 = a^3 - a^2x$
$\therefore a^2x = a^3$
$\boxed{\therefore x = a}$
So, x-intercepts is $(a, 0)$

Put $x = 0$
$\therefore 0 = a^3$ ⇒ Not possible

④ Asymptotes :-
$xy^2 = a^3 - a^2x$

(2) $x^3 + y^3 = 3axy$

→ Symmetric :-

The curve is symmetry about ~~y-axis~~, $y = x$ line. because interchanging x & y we get same eqn

→ Origin :-

Put $x = y = 0$

So, the curve is Passes through the origin

→ intercept :-

Put $x = 0$,

$y^3 = 0$

$\therefore y = 0$

Put $y = 0$

So, $x^3 = 0 \quad \therefore x = 0$

So, x-axis and y-axis are tangents at origin

$y = x$

$\therefore 2x^3 - 3ax^2 = 0$

$\therefore x^2(2x - 3a) = 0$

$\therefore x = 0 \quad$ & $\quad 2x = 3a$

$\therefore x = \frac{3a}{2}$

equating lowest degree terms too

$$-a^2 x = 0$$

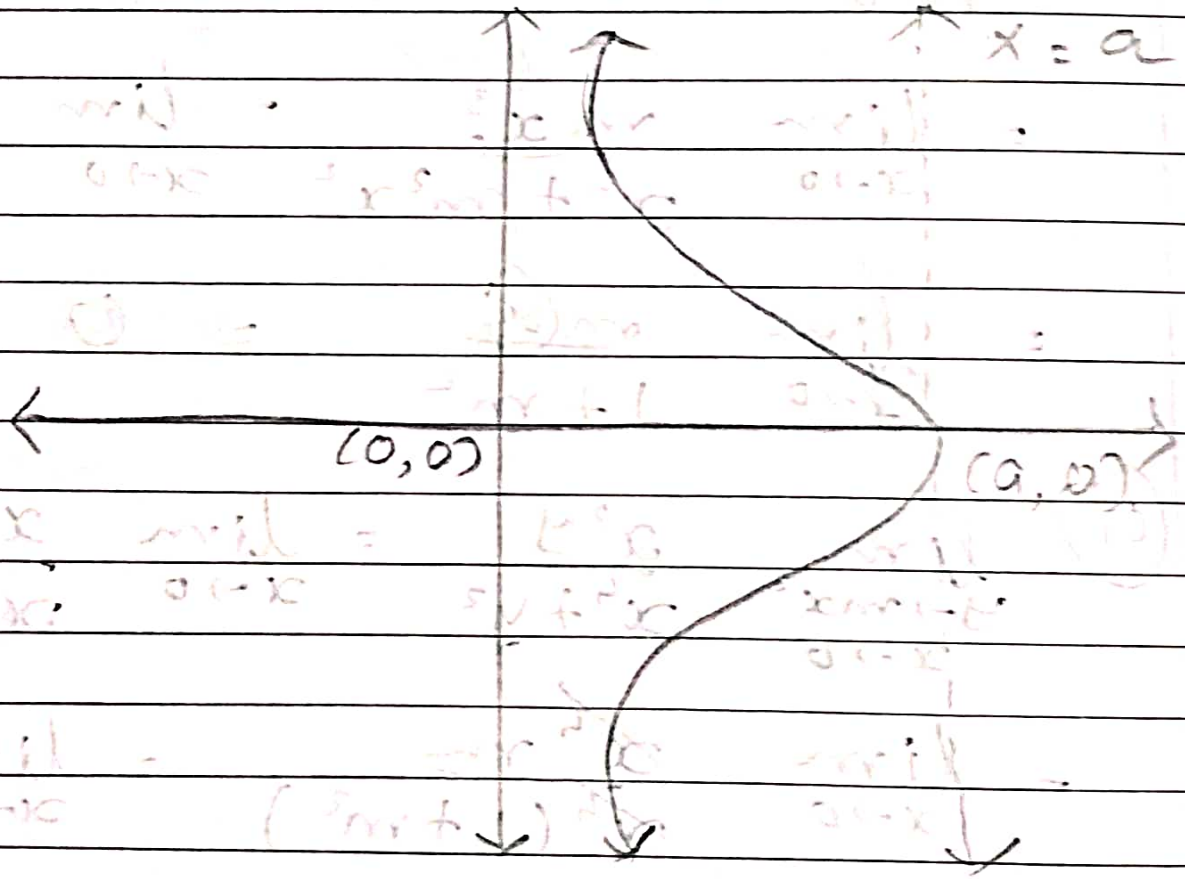
$$\therefore x = 0$$

∴ So, $x = 0$ is a asymptotes parallel to y-axis.

(5) Region:-

We get, $y^2 = \frac{a^3 - a^2 x}{x}$

$$y = \pm a \sqrt{\frac{a-x}{x}}$$



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→ The curve crosses the line $y = x$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ and $(0, 0)$.

⇒ Asymptotes :-

→ It has no asymptotes parallel to the axies.

$\phi_3(m) = 1+m^3$, $\phi_3(m) = 0 \Rightarrow m = -1$

$\phi_2 = -3am$

$$c = \frac{-\phi_2(m)}{\phi_3'(m)} = \frac{-(-3am)}{3m^2} = \frac{a}{m}$$

$\therefore c = -a$ when $m = -1$

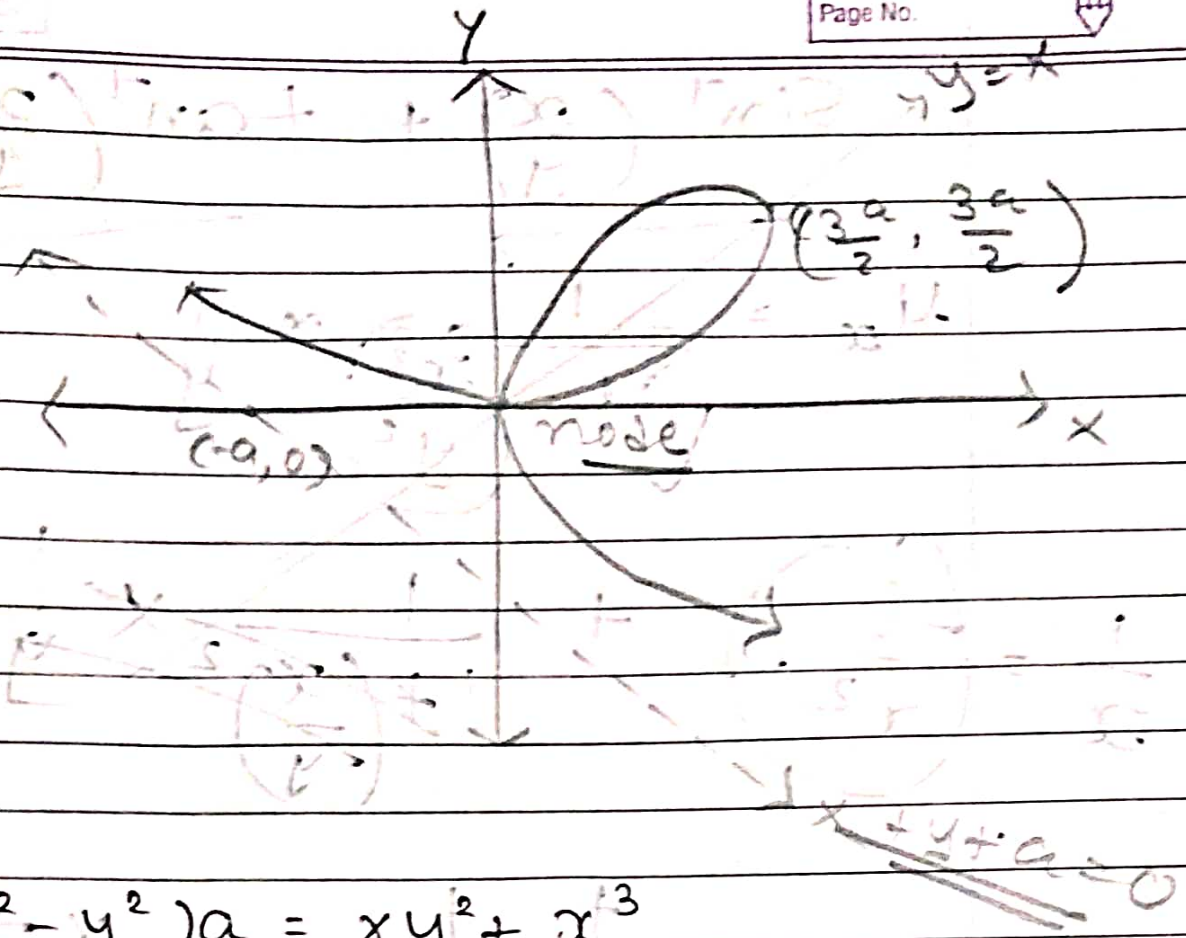
$\therefore y = mx + c$

$\therefore y = -x - a$

$\therefore x + y + a = 0$ is an asymptote.

→ Region

Curve does not lie in third quardrant.



(4) $(x^2 - y^2)a = xy^2 + x^3$

$x^2a - y^2a - xy^2 - x^3 = 0$

(1) Symmetry :-

The curve is symmetry about x-axis.

(2) origin :-

Put $x = 0$ $\quad x = y = 0$

we get $0 = 0$

$\therefore y^2/a = 0$

So, The Curve is Passes through the origin

③ Intercepts :-

Put $y = 0$

$\therefore x^3 = ax^2$

$\therefore x^3 - ax^2 = 0$

$\therefore x^2(x-a) = 0$

$\therefore x = 0 \quad \& \quad x = a$

So, x-intercepts is $(a,0)$ & $(0,0)$

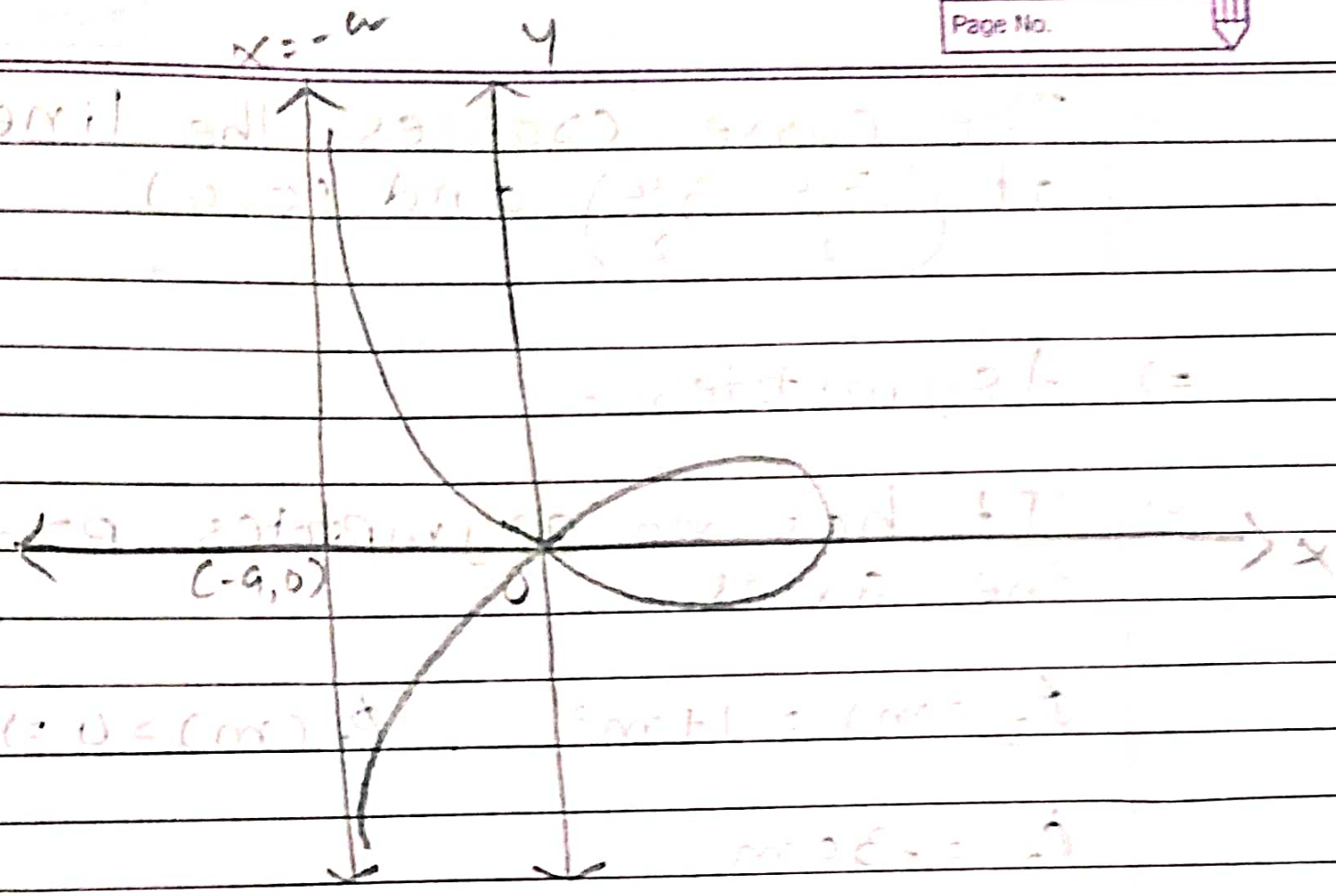
④ Asymptotes :-

There are no asymptotes parallel to x-axis.

No, the co-efficient of highest power of y is $(a+x)$.

So, the asymptotes parallel to y-axis is $\boxed{x = -a}$

⑤ Region :-

$$y = \pm \sqrt{\frac{a-x}{a+x}}$$

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